Problem-based learning in geometry courses: the impact on pre-service teachers

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ABSTRACT. A successful problem-based learning approach to the teaching of geometry in a content course for pre-service secondary teachers is presented. Based on a long term experience with this population a model for the students’ reactions to the pedagogy is offered. The implementation of a similar approach in a geometry content course for pre-service elementary teachers is documented with hard data. Obvious differences between two populations emerged. A need for further studies of the pedagogical approach for the two populations is suggested.

1. Introduction

Problem-based learning (PBL), is a relatively new approach to classroom practice (Gallagher, 1997). Barbara Duch of the University of Delaware offers the following definition of PBL:

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Problem-based learning is an instructional method that challenges students to “learn to learn,” working cooperatively in groups to seek solutions to real world problems. These problems are used to engage students’ curiosity and initiate learning the subject matter. PBL prepares students to think critically and analytically, and to find and use appropriate learning resources.

PBL was traditionally utilized in higher education in medical curricula, whose intrinsic multidisciplinary and multifaceted nature denounced the weaknesses of the traditional approach to teaching, based on lecture format. The literature on the effectiveness of the approach in the medical field is rapidly growing (Albanese and S. Mitchell, 1993), (Barrows, 2000).

In more recent years, PBL has found its way to different levels of the educational curriculum in the U.S. and in other countries, with various implementations in different disciplines (Blumberg, 2000), (Boud and Feletti, 1997), (Savin-Baden, 2000), (Schwartz et al., 2001), (Wilkerson and Gijsealers, 1996), (http://www.samford.edu/pbl/, http://www.udel.edu/pbl, http://www.imsa.edu/team/cpbl).

PBL in undergraduate and graduate education in mathematics has an illustrious and equally controversial forefather in R. L. Moore and his school at the University of Texas at Austin in the 1960s (Jones, 1977), (http://www.discovery.utexas.edu/rlm). A greatly modified version of the Moore method, more in line with the basic pedagogical principles
of contemporary PBL, has recently been championed by David Henderson at Cornell University (Henderson, 2001a), (Henderson et al., 1996). Several NSF grants (USE 9155873, DUE 9554651, DUE 9752807) allowed the Cornell group to hold Undergraduate Faculty Enhancement (UFE) workshops in which a PBL approach to the teaching of undergraduate geometry was presented.

It is a known fact, documented in literature (Schoenfeld, 1994), (Savin-Baden, 2000), that an implementation of PBL in a mathematics course has as a natural consequence a decrease in the content covered. The geometry classes are almost always terminal courses, making the issue of content coverage much easier to handle. For this reason the geometry classes are in a privileged position in the undergraduate mathematics curriculum for the implementation of PBL. Furthermore, a reduction of content in favor of deeper understanding is advocated in the Conference Board of the Mathematical Sciences (CBMS) report (CBMS, 2001), Chapter 1:

... college mathematics courses should be designed to prepare prospective teachers for the lifelong learning of mathematics, rather than to teach them all they would need to know in order to teach mathematics well.

As a result of his participation in one of the mentioned UFEs, the first author taught a geometry content course for pre-service secondary teachers for several semesters. The course integrated problem-based learning, intensive writing, and technology. The focus of the course
was the use of PBL to enhance the learning of the geometry content. Students were immersed in the pedagogy as a structural component of their learning experience, in line with the general recommendations of (CBMS, 2001). The pedagogical model was experienced but was never the object of meta discussions in class. Nonetheless, as an added bonus, after the first-hand exposure to the alternative pedagogy, several students started a process of reflection and re-evaluation of their pedagogical beliefs. This process, although intriguing and worth of further exploration, is not the focus of this work.

The positive experience and the ample anecdotal evidence of the success of the approach with pre-service secondary teachers motivated the first and third author to implement the same pedagogical approach for pre-service elementary teachers. The third unit in the sequence of mathematics content courses for pre-service elementary teachers, dedicated to elementary Euclidean geometry, was chosen for this work. With the help of a grant from the Indiana University SBC Fellows Program, the two authors’ goal was to gather hard evidence of the success of the approach. In this article we first present the pedagogical approach with pre-service secondary teachers and introduce a model describing students’ reactions. We then discuss the implementation of the pedagogy with pre-service elementary teachers and the instruments that were used to gather evidence for the students’ reactions model. Lastly we present the collected data and a statistical analysis conducted in collaboration with the second author. Noticeable differences between the two populations clearly emerged and are discussed in the paper.
2. PBL with pre-service secondary teachers

The most relevant rationale for the implementation of PBL and intensive writing in undergraduate mathematics is the concrete possibility of having the students experience, at an appropriate level, the process of authentic mathematics creation. A course in which definitions and theorems are not imposed from the outside as untouchable truths, but are commonly agreed upon, revised at different stages, and modified according to new contexts in which the mathematics maker is operating, makes the excitement of discovery a reality.

To accomplish this, each unit of the course for pre-service secondary teachers starts with the introduction of a geometry problem. The problems are designed to challenge known notions, are open-ended and exploratory in nature, and require the use of models (*Lenart® spheres*)\(^1\), manipulatives, or technology (*Geometer’s Sketchpad®*)\(^2\). Good examples of problems can be found in Henderson’s book (Henderson, 2001a). In order to give the reader a more concrete understanding of what a problem in this context might look like, we offer a specific example.

**PROBLEM:** What type of isometry is the result of the composition of two reflections?

As the ambient space in which the problem is posed is not specified, students have to address the same question in the plane (where they use Geometer’s Sketchpad), on the sphere (using the Lenart spherical

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\(^1\)Lenart Spheres are transparent, writable spherical models, a product of Key Curriculum Press, http://www.keypress.com/catalog/products/tools/Prod_LnrtSphre.html

\(^2\)Geometer’s Sketchpad is a dynamic geometry software package, also a product of Key Curriculum press, http: www.keypress.com/sketchpad
models), and on the cylinder (using appropriately constructed models and a properly developed notion of local isometry).

Students work in groups on the problem. The instructor listens and steers the discussion with carefully chosen questions. It is not uncommon for the students to raise and pursue issues which the teacher had not previously anticipated. Group work is followed by a discussion at the class level, which produces various solutions and/or opens more questions. Each student turns in a proposed written solution of the problem. A dialog between the instructor and each student ensues through repeated drafts of the assignment. When the class reaches a good understanding of the problem, a final draft is turned in and graded. The authors have a collection of anecdotal evidence of the excitement of the students. The open-ended nature of the problems utilized in the course also encourages a diversity of learners’ perspectives to flourish.

3. A model for students’ reactions

Four years of experience with pre-service secondary teachers in a geometry content course provided an opportunity to observe the reactions of the students to the pedagogy. Through successive refinements, a model for the students’ reaction was developed. This model was presented by the first author at the 1998 Cornell UFE where it was very positively received. Successively it appeared in print in (Henderson, 2001b).

We describe in detail the model below, and report a graphical representation for it in Figure 1.
Typically students come into this course not having been exposed to geometry since high school. Their memories of the high school experience are foggy, unpleasant, and carry the mark of the widespread attitude towards mathematics as a \textit{closed} subject, in which every question has one unique answer. This answer is believed to be reachable exclusively by a compulsory series of steps, which are supposed to be absorbed from the all-knowing teacher and memorized for future duplication, whether or not their meaning and rationale are clear. These recollections are experienced as expectations and they very quickly clash with the open-ended nature of the problems and the apparent lack of traditional guidance on the part of the teacher in the PBL model. As a result, students experience a great deal of frustration. The weaker
students are frustrated because they feel completely lost: “You are not teaching me.” The better students, the ones who are very skilled in figuring out the teacher’s expectations in a traditional classroom, are frustrated because of the lack of the traditional indicators of the teacher’s desiderata: “What do you want? Tell me what you want!” The frustration with the unfamiliar and challenging learning environment becomes a source of anger. Unfortunately some students get trapped in a vicious cycle in which frustration produces anger, which in turns feeds more frustration and more anger and so on. It is our experience that the majority of the students are able to move on from the frustration-anger trap, through an event that we like to describe as an epiphany. It assumes different forms for different students: it can be a successful group work session, a moment in which finally a particular problem “clicks”, a positive reinforcement from the instructor or from peers, a particularly fruitful writing session . . . A great sense of accomplishment, ownership and empowerment results from this experience and the majority of the students can then see how the pedagogy is beneficial to their learning experience.

The anecdotal evidence, collected through several semesters of students evaluations by the authors and by several other former participants to the UFE workshop, suggests that students often radically revise their attitudes and beliefs towards learning geometry and more generally towards learning mathematics, as an outcome of experiencing the alternative pedagogy. The need to go beyond anecdotal evidence is repeatedly voiced in the literature (Banta et al., 2001) and on the
mailing list teach-geom@cornell.edu, which connects UFE workshops participants. As briefly mentioned earlier, as byproduct of the experience we also witnessed several students who, greatly affected by the empowerment and ownership experienced during the semester, manifested the intention of implementing a similar pedagogy in their teaching. On the wave of their enthusiasm they often overlook the intrinsic problems and drawbacks of the methodology.

The validity of the above model asserts the success of a PBL approach, in all of its components. We strongly believe that our success as educators is fully realized if we can take a student to the level of the empowerment, notwithstanding the anger and the frustration experienced along the way.

The major goal of this research is consequently to test the validity and portability of the students’ reaction model. To ensure the largest possible population for data gathering we selected the geometry course for pre-service elementary teachers. Our choice is strongly supported by the Recommendations for Elementary Teacher Preparation in (CBMS, 2001):

\[ \ldots \text{the first priority of preservice mathematics programs must be to help prospective elementary teachers to rekindle their own powers of mathematical thought: with classroom experiences in which their ideas for solving problems are elicited and taken seriously, their} \]
sound reasoning affirmed, and their missteps challenged in ways that help them make sense of their errors.

We describe the implementation of PBL with the new population in the following section.

4. PBL with pre-service elementary teachers

The pre-service secondary teachers are mathematics majors by choice. This simple fact creates a general positive attitude towards the discipline. On the other hand, the pre-service elementary teachers do not explicitly declare their allegiance to the subject. Quite often they approach the mathematical component of their training with fear, hostility, and pre-existing frustration, see (CBMS, 2001), Chapter 1.

The attitudes toward mathematics in the context of the college experience of the second population were measured by a series of statements included in one of the instruments utilized in this research. The statements were constructed on a Likert-scale model from 7 (strongly agree) to 1 (strongly disagree). For the convenience of the reader, the relevant statements follow:

1. I have found my previous college classes to be challenging and intellectually stimulating.
2. I have found my previous mathematics college classes to be challenging and intellectually stimulating.
3. I have found my previous college classes enjoyable.
4 I have found my previous mathematics college classes enjoyable.

5 I remember the most important concepts that I learned in my previous college classes.

6 I remember the most important concepts that I learned in my previous mathematics college classes.

7 The most important concepts that I learned in my previous college classes are relevant to my life.

8 The most important concepts that I learned in my mathematics college classes are relevant to my life.

The box plots in Figure 2 clearly show a difference in attitude toward mathematics versus other disciplines.

Figure 2: Students Attitudes
Although students felt equally challenged by mathematics and other disciplines, the perceived level of relevance and their enjoyment of mathematics are significantly lower.

Our previous experiences and the above documented attitudes cautioned that a full blown implementation of the PBL model with the pre-service elementary teachers might fail. For this reason the approach is modified as follows:

i Problems are much smaller in scope;

ii Problems are concluded with solutions agreed upon at group/class level;

iii The repeated written drafts model for the assignments is only used for the final group project. (As a culmination of the class experience, a geometry topic from the elementary/middle school curriculum is selected by each group in cooperation with an area teacher interested in enriching his/her teaching with technology and a PBL approach. The project goes through several drafts which are presented in class for discussion and feedback. The final version is presented to the cooperating teacher.)

5. Statistical Analysis

The revised PBL approach described in the previous section was implemented in Mathematics for Elementary School Teachers III during the Spring and Fall 2001. The students completed three anonymous questionnaires at the beginning, in the middle, and at the end of the
semester. All questionnaires were modelled on a Likert scale from 7 (strongly agree) to 1 (strongly disagree). The middle-semester and end-semester questionnaires contained three statements that dealt directly with the previously outlined model for student reactions (Figure 1):

F I found the learning experience in this class frustrating.
A The learning experience in this class makes me angry.
E I have experienced deep satisfaction in investigating problems on my own in this class.

We looked at differences in scores of individual students from the end of a semester to the middle of a semester (27 subjects). The three variables FRUSTRATION, ANGER, and EMPOWERMENT corresponding to these three statements are studied (see Figure 3) to understand if there is enough support for the model.

Figure 3: Middle to end semester change in FRUSTRATION, ANGER, and EMPOWERMENT variables
The simple sum of changes in the FRUSTRATION variable from the middle to the end of the semester was +3; the mean changed from 4.96 in the middle of the semester to 5.07 at the end of the semester. The simple sum of changes in the ANGER variable from the middle to the end of the semester was +11; the mean changed from 3.30 to 3.78. The simple sum of changes in the EMPOWERMENT variable from the middle to the end of semester was + 4; the mean changed from 4.11 to 4.26.

The box plots in Figure 3 demonstrate the increase in both FRUSTRATION and ANGER variables from the middle-semester to the end of semester, indicating that the majority of students got trapped in the frustration-anger cycle. Only half of the students experienced increase in EMPOWERMENT variable after the problem-based approach. This was contrary to our expectations that the model for students’ reactions developed for pre-service secondary teachers would also apply to the new population.

In addition to the student’s reactions model analysis we were interested in the change of students’ opinions throughout the semester about the other aspects of problem-based learning approach, cooperative learning, and the use of technology.

To understand how these opinions changed during the course of a semester we looked at differences in scores of individual students from the end of a semester to the middle of a semester (27 subjects) and from the middle to the beginning of the semester (25 subjects). Note that the smaller group of subjects was not a subset of the bigger
Figure 4: Cochran-Mantel-Haenszel Statistics for PBL PEDAGOGY variable

group. We analyzed correlation using SAS Cochran-Mantel-Haenszel Statistic, which tests for association between the row specification and the column specification after adjusting for all other columns in the table. Due to the small sample size and the highly variable nature of this data, the study suffered from low statistical power. Groups were combined in order to maximize sample power under the expectation that the associations within groups would be similar. Consequently, low P-values were taken as supportive evidence of association.

The variable PBL PEDAGOGY consists of four original variables (note that the scale for two of the variables was reversed):

\textbf{PBL1 (Reversed scale)} I learn better when my instructor delivers all the necessary facts and sample problems.
PBL2 I learn better when I investigate and solve problems on my own.

PBL3 (Reversed scale) I expect my instructor to deliver all the necessary facts and sample problems.

PBL4 I expect to be asked to investigate and solve problems on my own.

Figure 5: Beginning to middle semester changes in PBL PEDAGOGY, COOPERATIVE LEARNING, and TECHNOLOGY variables

Cochran-Mantel-Haenszel Statistics for PBL PEDAGOGY (Figure 4) indicate the change in time. (For readability purpose we edited SAS output to contain new variables names.)

Even though this is not a statistically significant result, it is contrary to the expectation that our method will have a positive effect on students’ opinions about the problem-based learning approach. The
box plot in Figure 5 indicates negative change in individual students’ attitudes from the beginning to the middle semester while the box plot in Figure 6 indicates no change from the middle to the end of semester.

The simple sum of changes from the beginning to the middle semester was -5; +7 from the middle to the end of semester. A contrast between simple sums and box plots suggests large variations in individual student’s opinions from the middle to the end of semester.

Contrary to the previous results suggesting a decrease in positive attitude about the problem-based learning approach, an increase from the middle to the end of semester was found (Figure 7) on the two correlated variables:

SUP  *In this class I am learning only at a superficial level.*
LOSS I am at a complete loss in the learning environment created in this class.

Figure 7: Middle to end semester change in SUPERFICIAL LEARNING/COMPLETE LOSS variable

The variable COOPERATIVE LEARNING consists of three original variables:

CL1 I learn better in a collaborative (group) setting.

CL2 I feel comfortable to ask my peers for help when I need it.

CL3 I expect to be asked to work in a collaborative way with my peers.

Cochran-Mantel-Haenszel Statistics for COOPERATIVE LEARNING (Figure 8) indicate the change in time.
Figure 8: Cochran-Mantel-Haenszel Statistics for COOPERATIVE LEARNING variable

This supports the expectation that our pedagogy will have an effect on students’ opinions on a collaborative approach in learning mathematics. The box plot in Figure 5 indicates that the positive change occurred from the beginning to the middle of a semester while the Figure 6 indicates that the positive change was rolled back from the middle to the end of semester. The simple sum of changes from the beginning to the middle of a semester was + 6; - 4 from the middle to the end of semester. We would like to note that all of our students were commuter students that naturally experience difficulties in finishing collaborative projects.

The variable TECHNOLOGY consists of three original variables:

T1 I learn mathematics better with the help of technology.
T2 I expect my math teachers to use technology in the classroom.

T3 I expect to do assignments that require the use of technology.

Cochran-Mantel-Haenszel Statistics for TECHNOLOGY (Figure 9) indicate the change in time.

This supports the expectations that our approach will have a positive effect on students’ opinions on the use of technology in learning mathematics. The box plot in Figure 5 indicates that the positive change occurred from the beginning to the middle of a semester. The simple sum of changes from the beginning to the middle of a semester was +18; +6 from the middle to the end of a semester.

6. Conclusion

The geometry content course for pre-service secondary teachers described in section 2 successfully integrated problem-based learning, intensive writing, and technology. We have ample anecdotal evidence in support of the model for students’ reactions (Figure 1) to the pedagogy. The validity and portability of the students’ reactions model was tested in a geometry content course for pre-service elementary teachers. The statistical analysis in this study, along with free-form and numerical course evaluations, show that our students’ attitudes and beliefs about problem-based learning, intensive writing, and technology have changed. The hard evidence with the new population does not parallel our expectations nor prior experience with pre-service secondary teachers. We believe that the scaled down approach described in section 4 was responsible for the large number of students who got trapped in
the frustration-anger cycle and for the relatively poor showing of the empowerment phase of the proposed model. We recognize that we were overly cautious in implementing the full blown PBL approach with pre-service elementary teachers. This factor might be partially responsible for the clear differences in reactions of the two populations to the pedagogy. Pre-service elementary teachers were significantly more sensitive to the lack of direct instruction advocated by the model. This is illustrated by the contrasting comments collected from the two populations through anonymous teaching evaluation instruments.

<table>
<thead>
<tr>
<th>Pre-service secondary teachers</th>
<th>Pre-service elementary teachers</th>
</tr>
</thead>
<tbody>
<tr>
<td>I enjoyed the teaching style. It is one I hope to employ in my own classroom someday.</td>
<td>There was very little lecture. We were expected to work on material and figure it out in group, on our own.</td>
</tr>
<tr>
<td>I never believed that geometry could be a study where the student would be expected to use intuition and imagination...</td>
<td>More instruction would have been helpful.</td>
</tr>
<tr>
<td>I always thought there was only one way to prove a theorem and one very structured and traditional way to prove something.</td>
<td>More clear instruction.</td>
</tr>
</tbody>
</table>

We believe this sensitivity is partially due to our failure of providing these students with the same strong empowerment experience that the majority of pre-service secondary teachers received. Our next step is
then clear: we need to expose pre-service elementary teachers to the full blown PBL model. Further efforts in this direction will be pursued and documented.

Figure 9: Cochran-Mantel-Haenszel Statistics for TECHNOLOGY variable

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Alternative Hypothesis</th>
<th>DF</th>
<th>Value</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Nonzero Correlation</td>
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<td>2.4554</td>
<td>0.1171</td>
</tr>
<tr>
<td>2</td>
<td>Row Mean Scores Differ</td>
<td>2</td>
<td>3.9326</td>
<td>0.1400</td>
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<tr>
<td>3</td>
<td>General Association</td>
<td>28</td>
<td>24.3574</td>
<td>0.6625</td>
</tr>
</tbody>
</table>

Total Sample Size = 103

References


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REFERENCES


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