Chapter 2

THE VAN HIELE THEORY

*Let no one ignorant of geometry enter my doors*

... Plato (inscription he carved above the entrance to his academy).

This chapter reviews the literature concerning the van Hiele theory on the levels of understanding in geometry. The chapter introduces a number of important concepts and issues that need to be considered when success in geometry is to be evaluated. The chapter also summarises the background to van Hiele’s work, his levels of understanding, a comparison with the SOLO taxonomy, Piaget’s and Vygotsky’s work and the transition between the van Hiele levels.

Introduction

In Chapter 1 this researcher pointed out that teaching is an intervention in the learning of another person and that a necessary, though not sufficient, condition for intelligent teaching is to have a conscious theoretical model of the processes of human learning. It was also stated that Pierre van Hiele and Dina van Hiele-Geldof developed such a model.
Pierre van Hiele and Dina van Hiele-Geldof as teachers in Montessori secondary schools were concerned about the difficulties their students were having with their studies of geometry. It became apparent to them that secondary school geometry involved a high level of thinking and primary school geometry involved lower levels of thinking. After observation and discussion of their students’ progress, the van Hieles concluded that in learning geometry, the students seemed to progress through a sequence of five reasoning levels, from wholistic thinking to analytical thinking to rigorous mathematical deduction. They also concluded that to progress from one level to the next, students seemed to pass through five phases from an inquiry phase through to an integration phase.

The van Hieles supported this composite approach of levels and phases to teaching by defining the subject matter to be learned but at the same time defining the role of the teacher as a helper who guided the student through levels of understanding of the subject matter. Fuys, et al. (1988, p 4) stated “Their [the van Hieles’] research work focused on levels of thinking in geometry and the role of instruction in helping students move from one level to the next” and van Hiele (1986, p. 39) quoted from his 1955 work and stated: “The attainment of the new level cannot be effected by teaching, but still, by a suitable choice of exercises the teacher can create a situation for the pupil favourable to the attainment of the higher level of thinking.”

As a consequence of their observations the van Hieles challenged the current methods of teaching that solely involved the imparting of facts and methods which were
often not understood by students. They determined that teachers should concentrate on
the development of insight in their students helping them move from one level of
thinking to another higher level by learning *structures* rather than facts. Van Hiele
(1986), stated:

I had understood that the learning of facts could not be the purpose of
teaching mathematics, I was convinced that development of insight ought to
be the purpose. ... I learned that insight might be understood as the result of
perception of a structure (pp. 4-5).

**Structures**

According to van Hiele (1986):

Structure is an important phenomenon: It enables man and animal to act in
situations that are not exactly the same as those they have met before.
Structure saves man and animal from a never-ending life of trial and error.
Structure enables people to understand each other. People see the same
structure and they can express their harmony by continuing the structure in
the same way (p. 24).
Structures were considered to be *strong* or *feeble* by van Hiele depending on their rigidity. Strong structures were those that could only be extended in one way and hence could be continued with certainty whereas feeble structures were those that could only be continued with uncertainty with mistakes often being made. It should be noted here that van Hiele considered mathematical structures to be very rigid and therefore strong if the rule of the structure was given.

Van Hiele (1986) relied on Gestalt psychology to develop his ideas of structure and he believed that structures had four important properties. He (1986) stated:

1. It is possible to extend a structure. Whoever knows a part of the structure also knows the extension of it. The extension of a structure is subjected to the same rules as the given part of it.

2. A structure may be seen as a part of a finer structure. The original structure is not affected by this: the rules of the game are not changed, they are only enlarged. In this way it is possible to have more details take part in the building up of the structure.

3. A structure may be seen as a part of a more-inclusive structure. This more-inclusive structure also has more rules. Some of them define the original structure.
4. A given structure may be isomorphic with another structure. In this case the two structures are defined by rules that correspond with each other. So if you have studied the given structure, you also know how the other structure is built up (p. 28).

To clarify these four points van Hiele (1986) illustrated them with reference to the human skeleton. He pointed out that having looked at a skeleton “The extension of the structure may happen when we realise that we have such a skeleton ourselves” (p. 29). He suggests that a finer structure, as explained in point 2, is developed when we give names to parts of the skeleton and the third point occurs when “we begin to study skeletons of animals and to compare them with the human skeleton” (p. 29). Finally he suggests that point four is illustrated by the comparison of the skeleton of man with the skeleton of animals.

Van Hiele believed that the first and fourth properties of a structure are self-revealing and are innate in mankind whereas the second and third properties required study. He concluded that if education was to produce the development of insight then pupils should be stimulated to develop their recognition and use of the second and third properties of structure. Van Hiele argued that, if insight and hence understanding is to be achieved, it is essential that the student develops an understanding of the nature of structures.
Hence, the model of learning according to van Hiele is:

Perception of a Structure → Insight → Understanding

Therefore, the purpose of teaching should be the development of insight. It should be noted here that van Hiele (1986) summarised his concept of insight as follows:

1. Insight can be observed when there has been an adequate action in a new situation.

2. Insight can be ascertained when there has been action on the strength of an established structure from which the answers to new questions can be read.

3. The best examples of insight happen unexpectedly; they are not brought about by planning (p. 154).

But he insisted that intention must produce the adequate action in a new situation for insight to be gained.
The Van Hiele Levels

According to the van Hieles the student passes through five hierarchical levels of thinking. Originally the van Hieles numbered the levels basic or 0, and 1 to 4. Wirszup (1976) kept the five levels but renumbered the levels so that Level 0 became Level 1, Level 1 became Level 2 etc. The names used for the levels were first used by Hoffer (1979) as the van Hieles did not name the levels. In 1986 Pierre van Hiele started to use the 1 to 5 scale and consequently most researchers today use the same scale. As van Hiele was a teacher of mathematics he used examples from geometry to illustrate his levels though he did not restrict his theory to Mathematics.

It is important to note that when van Hiele was commenting on his 1955 work entitled “De Niveau’s in het Denken, Welke van Belang Zijn Bit het Onderwijs in de Meetkunde in de Cerste Klass van bet V.H.M.O.”, in his 1986 work, he stated that the “Tracing of levels of thinking that play a part in geometry is not a simple affair, for the levels are situated not in the subject matter but in the thinking of man” (p. 41). It should be understood here that there is some disagreement as to what the actual number of van Hiele levels should be. Hence, it is important for educators to understand how students think, the way in which brain functioning influences a student’s ability to learn and the rationale behind brain-based education.
Level 1 (Recognition)

The student operates on geometric figures, such as triangles, and parallel lines by identifying, naming and comparing them according to their appearance. Perception is visual only. A student who is reasoning at level 1 recognises certain shapes wholistically without paying attention to their component parts. For example, a rectangle may be recognised because it looks “like a door” and not because it has four straight sides and four right angles as there is no appreciation of these properties. Shape is important and figures can be identified by name.

Level 2 (Analysis)

The student discovers properties/rules of a class of shapes empirically, such as folding, measuring, analysing figures in terms of their components and relationships among components. At this level component parts and their attributes are used to describe and characterise figures. For example, a student who is reasoning analytically would say that a square has four “equal” sides and four “square” corners. The same student, however, might not believe that a figure can belong to several general classes and have several names, eg, the student may not accept that a rectangle is a parallelogram. A figure at this level presents as a totality of its properties. A student may be able to state a definition but will not have understanding.
**Level 3 (Ordering)**

By following or giving informal arguments the student logically interrelates previously discovered properties or rules. The student operates with these relationships both within a figure and between related figures. There are two general types of thinking at this level. Firstly a student understands abstract relationships among figures, e.g., the relationship between a rectangle and parallelogram and secondly a student can use deduction to justify observations made at level 2. The role of the definition and the ability to construct formal proofs are not understood at this level though there is a comprehension of the essence of geometry.

**Level 4 (Deduction)**

The student proves theorems deductively and establishes interrelationships among networks of theorems. The student can manipulate the relationships developed at level 3. The need to justify relationships is understood and sufficient definitions can be developed. Reasoning at this level includes the study of geometry as a formal mathematical system rather than a collection of shapes.
Level 5 (Rigour)

The student establishes theorems in different postulation systems and analyses and compares these systems. The study of geometry at level 5 is highly abstract and does not necessarily involve concrete or pictorial models. At this level the postulates or axioms themselves become the object of intense rigorous scrutiny. Abstraction is paramount.

According to the van Hieles, a learner passes through these levels when assisted by appropriate instructional experiences, and that a learner cannot achieve one level of thinking without passing through the previous levels. If a teacher tries to teach a student at one level when the student has not passed through the previous level the student will not understand the teacher and resort to rote learning. A teacher may try to present the new information at a lower level than is required by the information in order to assist students who are not operating at the appropriate level to learn. A teacher may also present the new information at a lower level than is required by the information due to the teacher’s own lack of knowledge of geometry. This is known as level-reduction which causes the student “to lose sight of the real relation between levels” (Structure and Insight, van Hiele, 1986, p. 53). Level reduction is a significant factor in the poor teaching of geometry and has incredible implications for teachers of geometry in particular and mathematics in general. It would appear that unless a teacher is aware of the van Hiele levels of learning and can recognise the levels at which their students are operating little real geometry will be taught or learnt in the classroom.
It is noteworthy that in van Hiele’s 1986 work he refers to there being three levels, two levels and five or more levels. He also refers to an alternative set of three levels. Teppo (1991) suggests that Pierre van Hiele currently characterises his model in terms of three rather than five levels and Lawrie (1998) stated that “The existence of this latter model of only three levels was confirmed in personal communications in 1994 with Dr van Hiele at the Hague and again at the University of New England, Armidale” (p. 9). The three latter model levels with the 1 to 5 level system in parentheses are Visual (Level 1), Descriptive (Level 2 and Level 3) and Theoretical (Level 4 and higher). Pegg and Davey’s (1998) description of these three levels is favoured by Lawrie (1998) over that of Fuys, et al. (1988), as she felt that when the composition of the original levels is examined with regard to the understanding of geometry, Pegg and Davey’s approach was more logical. Pegg and Davey’s (1998) description of the three levels of the alternative model are:

Visual Level: decisions are guided by a visual network.

Descriptive Level: the elements and relations are described.

Theoretical Level: deductive coherence is prominent; geometry generated according to Euclid is considered.
The van Hieles (1958) identified properties of the levels to which Usiskin gave the names adjacency, distinction and separation. Inherent in the van Hiele theory is the belief that in understanding geometry a person must go through the levels in order. Hence Usiskin added a fourth property which he named fixed sequence. He (1982) stated:

It is inherent in the van Hiele theory that, in understanding geometry, a person must go through the levels in order. We call this the fixed sequence property of the levels.

Property 1: (fixed sequence) A student cannot be at van Hiele level \( n \) without having gone through level \( n-1 \).

Property 2: (adjacency) At each level of thought what was intrinsic in the preceding level becomes extrinsic in the current level.

Property 3: (distinction) Each level has its own linguistic symbols and its own network of relationships connecting those symbols.

Property 4: (separation) Two persons who reason at different levels cannot understand each other (pp. 4-5).
Whilst the interest in the categorisation of the levels is ongoing it is important to note that there is a consensus of opinion amongst researchers that the levels exist and are hierarchical and that they do measure cognitive development. However most of the ongoing research uses the original five levels renumbered 1 to 5.

Summarising the preceding information, it appears that the major characteristics of the van Hiele levels are:

- the levels are sequential
- each level has its own set of symbols, network of relations and terminology
- what is implicit at one level becomes explicit at the next
- material taught to students above their level is subjected by them to a reduction of level
- progress from one level to the next is more dependent on instructional experience than on age or maturation
- one goes through various “phases” in proceeding from one level to the next.

**Phases that Lead to a Higher Level**

The van Hieles defined five phases in the learning process through which they believed students must pass before “jumping” to the next level. They believed the levels were discrete, stating:
The discontinuities are ... jumps in the learning curve, these jumps reveal the presence of levels. The learning process has stopped; later on it will start itself once again. In the meantime, the pupil seems to have “matured”. The teacher does not succeed in further explanation of the subject. He and ... the other students who have reached the new level seem to speak a language which cannot be understood by the pupils who have not yet reached the new level. They might accept the explanation of the teacher, but the subject taught will not sink into their minds. The pupil himself feels helpless; perhaps he can imitate certain actions, but he has no view of his own activity until he has reached the new level. At the time the learning process will take on a more continuous character. Routines will be formed and an algorithmic skill will be acquired as the prerequisites to a new jump which may lead to a still higher level (1958, pp. 75-76).

Fuys, et al. (1988) paraphrased van Hiele-Geldof’s original definition of the phases and gave examples, stating:

According to van Hiele (1955/1986), progress from one level to the next involves five phases: information, guided orientation, explicitation, free orientation, and integration. The phases which lead to a higher level of thought, are described as follows with examples given for transition from level 1 to level 2.
**Information or Inquiry:** The student gets acquainted with the working domain (e.g. examines examples and non-examples).

**Guided Orientation:** The student does tasks involving different relations of the network that is to be formed (e.g. folding, measuring, looking for symmetry).

**Explicitation:** The student becomes conscious of the relations, tries to express them in words, and learns technical language which accompanies the subject matter (e.g. expresses ideas about properties of figures).

**Free Orientation:** The student learns, by doing more complex tasks, to find his/her own way in the network of relations (e.g. knowing properties of one kind of shape, investigates these properties for a new shape, such as kites).

**Integration:** The student summarizes all that he/she has learned about the subject, then reflects on his/her actions and obtains an overview of the newly formed network of relations now available (e.g. properties of a figure are summarized) (pp. 7-8).
According to van Hiele, memorisation occurs at the end of the fifth phase and ordinary learning occurs. He believes that whilst the attainment of a new level cannot be effected by teaching, a teacher can create, by a careful choice of exercises, an environment that enables the student to attain the higher level of thinking. He went on to say that “You can say somebody has attained a higher level of thinking when a new order of thinking enables him, with regard to certain operations, to apply these operations on new objects” (1955, p. 289).

It appears that many researchers (Pegg & Davey, 1991; Frykholm, 1994; Whitman, 1994; Ahuja, 1996; Lawrie, 1998) believe that in the learning of geometry the van Hiele levels are useful. However, the research on the levels is aimed at the students and is often an attempt to verify van Hiele’s work and to draw conclusions on the implications for curriculum planning. Van Hiele states that “The transition from one level to the following is not a natural process; it takes place under the influence of a teaching-learning program” (1986, p. 50). Teachers hold the key to this transition from one level to the next. Unfortunately, Brodie (1992) showed that primary school teachers in one region of NSW are generally not aware of van Hiele’s research and that they dislike geometry. It is reasonable to suggest that the lack of knowledge about the van Hiele levels and the dislike of geometry by teachers contribute to poor student achievement.
Piaget, Vygotsky and van Hiele

Piaget (1896 - 1980) was a genetic epistemologist who was the first to introduce the concept of levels of learning. He believed that progress from one level to the next was due to biological changes and that the higher level was innate and was attained when students became aware of it. According to Piaget (1970), mathematical structure was the basis of, and defined, the whole structure underlying cognitive development. Piaget’s four levels of development are: Sensorimotor (0-2 years); Preoperational (2-7 years); Concrete operational (7-11 years) and Formal operational (11-adult).

Vygotsky (1896 - 1934) was a contemporary of Piaget. In an attempt to understand cognitive processes, Vygotsky tried to work out the formation of intellect by focussing on its process of development. He concluded that individual intellectual development could not be understood without reference to the social and cultural context within which the development occurs. Vygotsky stressed the importance of language in learning development. He did not focus on stages of development like Piaget but rather focussed on development throughout life and, in doing so, was perhaps the first advocate of lifelong learning.

There has been much debate about the relationship between the ideas of Piaget and Vygotsky. Generally speaking, the debate centres on the fact that Piaget believed that children constructed knowledge through their actions on the world whereas Vygotsky claimed that understanding was social in origin. Cole and Wertsch (1996)
believe that “in principle, Piaget did not deny the co-equal role of the social world in the construction of knowledge” (p. 1) and that “Vygotsky, contrary to another stereotype, insisted on the centrality of the active construction of knowledge” (p. 2). They concluded that the major difference between the work of Piaget and Vygotsky did not lie in the sociogenesis of mind but in the role that cultural artifacts play in constituting the two poles of the individual-social antinomy. Vygotsky sees the artifacts playing a central role in determining what and where the mind is and in doing so focuses on issues that do not have any corresponding concepts in the work of Piaget.

Even though van Hiele (1986) stated: “an important part of the roots of my work can be found in the theories of Piaget” (p. 5), he disagreed with much of Piaget’s theory. Van Hiele believed that Piaget’s psychology was one of development and not one of learning and he was concerned that Piaget’s two levels (preoperational and concrete operational) could not accommodate learning in geometry, which according to him, required more than two levels of understanding. Indeed, van Hiele (1986), suggested that “Some of Piaget’s results would have been more intelligible if he had distinguished more than two levels” (p. 5). The fact that Piaget did not acknowledge the important role that language (as did Vygotsky) played in moving from one level to another, was also of concern to van Hiele.

Van Hiele was particularly concerned that Piaget did not understand that structures of a higher level were the result of study of the lower level. In van Hiele’s theory if a structure, which is a given thing obeying certain laws, was a strong structure
it was usually possible to superimpose a mathematical structure onto it whereas in Piaget’s theory the mathematical structure always defined the whole structure.

Piaget believed that children were born with the higher structure and needed only to become aware of it whereas van Hiele believed that the rules of the lower level became the structure of the higher level. For van Hiele structure is everything. However, it was his interest in the work of Piaget that ultimately led him to identify the role of language in learning, levels of thinking and the way in which students moved from one level of understanding to the next.

Whilst there has been much comment on the relationship between Piaget’s and Vygotsky’s work (Cole & Wertsch, 1996; Nicholl, 2002) and some comment on the relationship between Piaget’s and van Hiele’s work (Brodie, 1992) there appears to be no research on the relationship between van Hiele’s and Vygotsky’s work. Van Hiele believed that cognitive growth occurs through a series of stages or levels whereas Vygotsky did not. However, both van Hiele and Vygotsky accepted the importance of language in the development of the intellect. Van Hiele (1986) stated “Piaget did not see the very important role of language in moving from one level to the next” and Vygotsky (1978) stated “the child begins to perceive the world not only through its eyes but also through its speech. And later it is not just seeing but acting that becomes informed by words” (p. 32).
Similarly, van Hiele and Vygotsky, unlike Piaget, accepted the need for teaching to improve the learning of a student. Van Hiele (1986) stated “The transition from one level to the following is not a natural process; it takes place under influence of a teaching-learning program” (p. 50) and Nicholl (2002) stated “Vygotsky's approach of scaffolding and guided discovery suggests that a guiding hand by the teacher is critical for effective learning”.

Van Hiele and Vygotsky were both contemporaries of Piaget and all three were constructivists. Van Hiele, and Vygotsky saw the role of the teacher and the role of language in the construction of knowledge by an individual as crucial while Piaget did not. However, in a Piagetian classroom the teacher's role is to provide a rich environment for the spontaneous exploration of the child. Piaget and van Hiele believed that individuals passed through levels of learning but Vygotsky did not. Piaget did not consider society or its culture to be crucial to learning but Vygotsky considered it essential.

The Structure of the Observed Learning Outcome (SOLO) Taxonomy

Whilst the work of Piaget, Vygotsky and van Hiele needs to be considered when determining the best approach to the teaching of Geometry it should be noted that Pegg (perhaps the most knowledgeable researcher in Australia on van Hiele's work) and Davey (1989), determined that a comparison between the level descriptors of the van
Hiele theory and the SOLO taxonomy should be researched. They used descriptions of common two-dimensional geometric shapes written by students in grades three to seven as the basis for the comparison. Before commenting on the results a brief look at the SOLO taxonomy is appropriate.

Biggs and Collis (1982) first described the SOLO taxonomy. In 1986 Courtney suggested that the SOLO taxonomy was “a five level hierarchy which was designed to help teachers evaluate the quality of students’ thinking” (p. 47). The stages of the taxonomy, together with the age at which the SOLO modes can be expected to emerge given an appropriate teaching/learning environment, are:

1. **Prestructural**: pre-operational developmental base stage - 4-6 years old.

2. **Unistructural**: early concrete developmental base stage - 7-9 years old.

3. **Multistructural**: middle concrete developmental base stage - 10-12 years old.

4. **Relational**: concrete generalisation developmental base stage - 13-15 years old.

5. **Extended Abstract**: formal operations developmental base stage - 16+ years old.
Each of the levels is defined in terms of capacity, relating operation and consistency, and closure. Biggs and Collis (1982) stated:

**Capacity** ... refers to the amount of working memory or attention span ...

**Relating Operation** ... refers to the way in which the cue and the response interrelate ... [and] **Consistency and Closure** ... refer to two opposing needs felt by the learner: one is the need to come to a conclusion (to close) and the other is to make consistent conclusions so that there is no contradiction either between the conclusion and the data, or between different possible conclusions (pp. 26-27).

The van Hiele theory explores the manner in which students develop understanding of a topic whereas the SOLO taxonomy provides a method of evaluating the learning that has taken place.

According to Pegg and Davey (1989), however, the results of their research indicated that the SOLO taxonomy more accurately described the quality of student thinking when compared to the van Hiele theory. This conclusion resulted from Pegg’s belief that “The SOLO taxonomy identifies the concentration on ‘one aspect’ as unistructural where the van Hiele theory does not” (p. 25) and because Pegg believed that it was possible that van Hiele saw the identifying of one property and of many properties as horizontal growth whereas his research indicated that “there was a ‘vertical’ growth associated with the move from identifying one property to that of
identifying many properties of a figure” (p. 25). However, a consideration of van Hiele’s work, indicates that he defined five phases in the learning process through which he believed students must pass to move from one level to the next.

Courtney (1986) concluded that the SOLO taxonomy had broad curriculum applicability and could make a substantial contribution to improved teaching and learning. While his work does not specifically refer to geometry (it actually was written for the Australian Geography Teachers Association) it complements the research by Pegg and Davey as he saw the SOLO taxonomy as relevant to all teachers. Biggs and Collis (1982) devote a chapter to the SOLO taxonomy and mathematics. Whilst not specifically mentioning geometry, they stated:

...it would appear reasonable that in content-process areas we would expect mainly unistructural responses in the early years of elementary school, multistructural in the later years of elementary school, relational in early and middle high school, and extended abstract only from those at the upper levels of high school who have chosen to put a lot of effort into mathematics. ... The teacher faced with a particular student at a particular time working on a specific problem needs to be able to ascertain the real level of functioning and work from that point and not subsume the individual child under the general rubric (pp. 90-91).
Units of Work Based on van Hiele’s Levels

There have been attempts to develop units of work based on van Hiele levels but the validity of the units has not been tested. Flores (1993) developed a unit of work on Pythagoras’ theorem in the context of the van Hiele levels. He showed that at each of the van Hiele levels it was possible to develop an understanding of Pythagoras’ theorem. As the complexity of the application of Pythagoras’ theorem increased, the higher the van Hiele level needed to understand the application, for example, at van Hiele level 1 (Recognition), Flores illustrated the theorem with tangrams and at van Hiele level 4 (Deduction), he used the proof of Euclid’s Proposition 47. Craine and Rubenstein (1993) produced a hierarchical structure of quadrilaterals to illustrate the learning of a geometric concept by moving from the van Hiele levels of visualisation and analysis through the level of informal deduction to the level of formal deduction. They developed a quadrilateral hierarchy chart that required van Hiele level 1 understanding and extended the chart using understanding associated with each van Hiele level so that the properties of the quadrilaterals were discovered or recognised by the students and the students’ concept of definition and its role in a formal system was expanded and strengthened. Pegg and Davey (1991) produced three activities; descriptions, minimum properties, and class inclusion to assess the van Hiele level of students’ geometric understanding. They found that students provided different descriptions as they grew in geometric understanding and that the activities they developed could be used to determine a student’s van Hiele level of understanding and as such were a useful tool for teachers.
With increased student access to computers, software based on the van Hiele levels is being developed. In America the second edition of the book *Discovering Geometry* (Serra, 1997) is based on the van Hiele levels in keeping with the *Curriculum and Evaluation Standards for School Mathematics* (1989) developed by the National Council of Teachers of Mathematics. It leads students to discover and master concepts and relationships before they are introduced to formal proofs and it makes use of software developed for use with the book.

**Van Hiele Levels and Three-Dimensional Geometry**

Saads and Davis (1996) investigated the van Hiele levels using three-dimensional geometry and the spatial abilities of a group of pre-service secondary teachers in Southampton, England. They gave a written test to 25 students enrolled in secondary initial teacher training. The test consisted of seven questions containing sub-questions and was designed to access Del Grande’s spatial perception categories. Del Grande (1987) suggested that associated with geometric understanding is a developing sense of general spatial perception. He proposed seven spatial abilities that seemed to be of greatest relevance in academic development in geometry. Five of these: perceptual constancy, figure ground perception, position in space perception, visual discrimination and spatial relationships were used in Saads and Davis’ test. The test reliability was measured using the Kuder-Richardson inter-term reliability method. This
method produces a reliability coefficient between 0 and 1 for each item. As the reliability coefficient for question 5 was very low it was removed from the test scores.

A modification of the Gutiérrez, Jaime and Fortuny (1991) coding system was used by Saads and Davis to construct a table of the results. These results showed that the van Hiele levels could be applied to 3-dimensional geometry, supported the hierarchical structure of the van Hiele levels and supported the non-hierarchical structure of the Del Grande Spatial Perception categories.

This research is interesting to the researcher as it linked the van Hiele levels of geometry with preservice teachers and because it indicates that a van Hiele level test may be coded in more than one way. Hence, the coding to use in the present research will have to be determined after careful consideration of the research available for each of the coding methods.

The Transition between the van Hiele Levels

Gutiérrez, et al. (1991) concluded that the van Hiele levels were not discrete and suggested a way of identifying students who were in transition between the van Hiele levels. They based their conclusion on the fact that:
Although most students show a dominant level of thinking when answering open-ended questions, a large number of them clearly reflect in their answers the presence of other levels, and there are some students whose answers show two consecutive dominant levels of reasoning simultaneously (p. 237).

In support of their conclusion they quoted the work of Fuys, et al. (1988) who assigned a student to Level 1-2 to indicate that the student used both levels of reasoning for a certain activity.

Gutiérrez, et al. (1991) suggested that the acquisition of a specific van Hiele level did not happen instantaneously but rather took months or even years to attain. They quantified the acquisition by using a scale of 0 to 100 divided into five periods which were characterised by the qualitatively different ways in which the students reasoned. Below is a representation of the scale. It should be noted that the values assigned to the limits by the researchers were subjective.

Table 2.1:

*Degrees of Acquisition of a van Hiele Level according to Gutiérrez, Jaime & Fortuny (1991)*

<table>
<thead>
<tr>
<th>Period</th>
<th>Range</th>
</tr>
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<tbody>
<tr>
<td>No Acquisition</td>
<td>0 to 15</td>
</tr>
<tr>
<td>Low Acquisition</td>
<td>&gt;15 to 40</td>
</tr>
<tr>
<td>Intermediate Acquisition</td>
<td>&gt;40 to 60</td>
</tr>
<tr>
<td>High Acquisition</td>
<td>&gt;60 to 85</td>
</tr>
<tr>
<td>Complete Acquisition</td>
<td>&gt;85 to 100</td>
</tr>
</tbody>
</table>
To explain their conclusion, Gutiérrez et al. (1991) reasoned that initially students were not conscious of the thinking methods specific to a new level and hence had *No Acquisition* of this level of reasoning but once students were aware of the methods of thinking at a particular level they tried to use them but failed due to their lack of experience and hence returned to the lower level. Such students were said to have *Low Acquisition* of the level. They believed that the students then progressed through an *Intermediate and High Level of Acquisition* as their experience grew until the students attained *Complete Acquisition* of the new level at which time they had complete mastery of this way of thinking and used it without difficulty.

To assign students to a specific degree of acquisition of a level the researchers assessed the students using a series of open-ended items and criteria for evaluating their responses to each item. For each item the student’s response was assigned a score related to the acquisition scale. The criteria were divided by Gutiérrez et al. (1991) into eight types as in Table 2.2 below.
Table 2.2:

*Degrees of Acquisition of a van Hiele Level Scale according to Gutiérrez, Jaime & Fortuny (1991)*

<table>
<thead>
<tr>
<th>Type</th>
<th>Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type 0</td>
<td>No reply or answers that cannot be codified</td>
</tr>
<tr>
<td>Type 1</td>
<td>Answers that indicate that the learner has not attained a given level but that give no information about any lower level</td>
</tr>
<tr>
<td>Type 2</td>
<td>Wrong and insufficiently worked out answers that give some indication of a given level of reasoning: answers that contain incorrect and reduced explanations, reasoning processes or results.</td>
</tr>
<tr>
<td>Type 3</td>
<td>Correct but insufficiently worked out answers that give some indication of a given level of reasoning: answers that contain very few explanations, inchoate reasoning processes, or very incomplete results.</td>
</tr>
<tr>
<td>Type 4</td>
<td>Correct or incorrect answers that clearly reflect characteristic features of two consecutive van Hiele levels and that contain clear reasoning processes and sufficient justifications.</td>
</tr>
<tr>
<td>Type 5</td>
<td>Incorrect answers that clearly reflect a level of reasoning; answers that present reasoning processes that are complete but incorrect or answers that present correct reasoning processes that do not lead to the solution of the stated problem.</td>
</tr>
<tr>
<td>Type 6</td>
<td>Correct answers that clearly reflect a given level of reasoning but that are incomplete or insufficiently justified.</td>
</tr>
<tr>
<td>Type 7</td>
<td>Correct, complete, and sufficiently justified answers that clearly reflect a given level of reasoning</td>
</tr>
</tbody>
</table>
The weightings from the “Degrees of Acquisition of a van Hiele Level Scale” were assigned as in table 2.3 below.

Table 2.3:

<table>
<thead>
<tr>
<th>Type</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>25</td>
</tr>
<tr>
<td>4</td>
<td>50</td>
</tr>
<tr>
<td>5</td>
<td>75</td>
</tr>
<tr>
<td>6</td>
<td>80</td>
</tr>
<tr>
<td>7</td>
<td>100</td>
</tr>
</tbody>
</table>

The open-ended test items were produced using 3-dimensional geometry and were related to a particular van Hiele level. A vector (level, type) was assigned to each answer where “level” is the van Hiele level and “type” is the type of answer. If “type” was zero, “level” was considered empty. The degree of acquisition of a van Hiele level by a student was determined by calculating the arithmetic average of the weights of the vectors for all the items that could have been answered at that level.

The subjective application of a rating to the acquisition scale and its subsequent use in the calculating of an arithmetic average is questionable. Earlier it was noted that
van Hiele suggested that progress from one level to the next involved moving through five phases. It is possible that the five periods of Gutiérrez et al. (1991) are just another way of viewing van Hiele’s five phases. The need for the *complete acquisition* stage to be reached before a student is classified as operating at the higher level supports this assumption. It could be concluded, therefore, that this research does not necessarily discredit the discreteness of the van Hiele levels but rather reinforces the van Hiele theory.

**Comments**

The van Hiele levels have generally been accepted as a reasonable explanation as to how students learn geometry. The van Hiele theory, originally developed using 2-dimensional Euclidian geometry has been shown to apply to other areas of geometry. Whilst there is some disagreement on the number of levels and what they should be named the last or highest level has proven to be difficult, or impossible, to measure in a test.

It appears generally accepted by educationalists and psychologists that students pass through phases as they pass from one van Hiele level to another. Research and analysis of the phases was completed by de Block-Docq (1992) in a doctoral thesis in which she replicated Dina van Hiele-Geldofs lessons.
Units of work based on the van Hiele levels have proved useful to students and teachers. More use of software seems to hold the key to meeting the needs of some of the new curriculums produced both in Australia and overseas.

As indicated by Pegg and Davey (1989), the SOLO taxonomy continues to be useful to teachers of geometry. Their research on the overlap of van Hiele’s levels with the taxonomy, however, appears to be the only research that has been done in this area.

The work of Piaget and van Hiele suggests that students proceed through levels of understanding. Van Hiele believes that Piaget’s first two levels are not sufficient to explain the way in which students learn geometry. Vygotsky and van Hiele believe that language and teacher intervention is crucial to learning.

Fuys, et al. (1988) suggested that the transition by a student from one van Hiele level to another involved the student passing through five periods. Van Hiele suggested that students pass through five phases as they move from one level to another. It is possible that the five periods of Fuys, et al. (1988) are just another way of viewing van Hiele’s five phases.
Conclusion

Van Hiele believes that teacher intervention is crucial to student achievement of levels of understanding in geometry. He also believes that if individuals operate at different levels that ineffective rote learning occurs. Pegg and Davey believe that the van Hiele levels are basic to improving the teaching of geometry. Therefore, it is important then, that preservice teachers have an understanding of van Hiele’s model and are aware of their level of understanding in geometry. This would allow them to teach effectively the subject matter by guiding their students through the phases of each level and through the levels themselves. Failure to do this would result, according to van Hiele, in rote learning in which students memorise the right answer without understanding.

As noted before (see Chapter 1), teachers must know and understand the mathematics that they are teaching if they are to be effective teachers. According to van Hiele, it is only at level 3 that an individual has a comprehension of the essence of geometry. Therefore, to teach geometry effectively, a teacher needs to be at least at van Hiele level 3. Hence, it is important to determine the van Hiele level of the preservice teachers in this research. This data may lead to a reduction in the current apprehension about geometry in teachers and their students. Such a reduction may lead to more success in geometry in Australian school students and an improvement in Australia’s knowledge economy.
Van Hiele believes that level of understanding in geometry is not related to a particular age even though he believed that age was important to understanding. He stated (1986): “It would, however, be a deplorable error to suppose that a level is attained as the result of a biological maturation” (p. 65) and “The age of the children is important, in so far as they must have had sufficient time to go through the necessary learning processes” (p. 65).

Hence, the relationship between van Hiele level and age should also be examined as the result of this research may indicate the need to change current teaching methods in geometry.

These conclusions generate the following research questions:

1. What is the van Hiele level of preservice teachers?

2. What affect does gender, age and education have on the acquisition of geometric knowledge by an individual?